BMAN 70381: Foundations of Finance Theory: Exercises

September 2009

1. Mean-variance portfolio analysis:

   Assume the following data for stocks 1 and 2:

   (a) Expected cash flow: $E(x_1) = 6$, Standard deviation: $\sigma_1 = 1.5$, Stock value: $S_1 = 5$.

   (b) Expected cash flow: $E(x_2) = 2.5$, Standard deviation: $\sigma_2 = 0.4$, Stock value: $S_2 = 2.2$. Correlation between the cash flows: $\rho_{1,2} = 0.4$.

   Assume that the risk-free rate of interest is $r_f = 0.04$.

   Assume a mean-variance utility function. Also, assume the following for investors 1 and 2: wealth, $w_1 = 5$, $w_2 = 12$. Risk parameter, $\lambda_1 = 0.3$, $\lambda_2 = 1$.

   (a) State the first order conditions for a maximum of the utility of investor 1.

   (b) Compute the optimal stock portfolios for investors 1 and 2 and the amount of borrowing/lending.

   (c) Explain which investor is the more risk averse

   Note, given a matrix:

$$A = \begin{pmatrix} a, b \\ c, d \end{pmatrix}$$

   the inverse matrix is

$$A^{-1} = \begin{pmatrix} \frac{d}{ad-bc}, \frac{b}{bc-ad} \\ \frac{c}{bc-ad}, \frac{a}{ad-bc} \end{pmatrix}$$

2. Mean-variance portfolio analysis and the CAPM

j	$E(x_j)$	s_j	σ_j	$\rho 1, j$	$\rho 2, j$	$\rho 3, j$	N_j
1	9.2	7	1.6	1	0.5	0.3	10,000
2	37	33.5	2	0.5	1	0.7	5,000
3	20	18	1	0.3	0.7	1	12,000

i	λ_i	w_i
1	2.5	11
2	0.5	20

Given the above data for 1-period cash flows and a risk-free rate of 5%.

(a) Calculate the optimal portfolios of shares for investors 1 and 2.

(b) Suggest changes in stock prices that would lead to equilibrium assuming these are the only investors.

3. The Black model

   Assume you wish to value a 1-year call option on a stock given:

   Stock price \$91

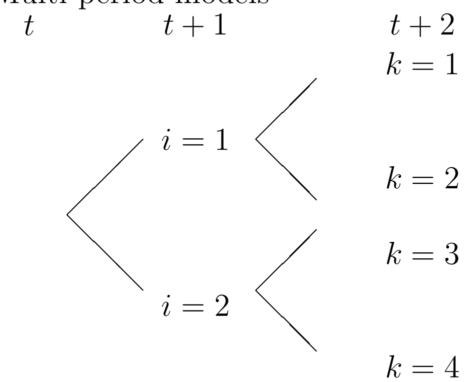
   Strike price \$110

   volatility 25%

   Risk free rate (continuously compounded) 6%

   What is the forward value of the option assuming a) no dividends on the stock? and b) a dividend yield of 3%? What is the spot value of the option in both cases?

4. Multi-period models



Let the spot state prices, $q_i^* = q_i B_{t,t+1}$ and $q_k^* = q_k B_{t,t+2}$

$$q_i^* = \begin{bmatrix} q_{i=1}^* \\ q_{i=2}^* \end{bmatrix} = \begin{bmatrix} 0.4 \\ 0.5 \end{bmatrix}$$

$$q_k^* = \begin{bmatrix} q_{k=1}^* \\ q_{k=2}^* \\ q_{k=3}^* \\ q_{k=4}^* \end{bmatrix} = \begin{bmatrix} 0.16 \\ 0.19 \\ 0.19 \\ 0.19 \end{bmatrix}$$

(a) compute the bond prices $B_{t,t+1}$ and $B_{t,t+2}$

(b) compute the conditional (forward) state prices $q_{i,k}$ and the conditional bond prices $B_{t+1,t+2,i}$

(c) If the payoff on an asset at $t+2$ is given by the vector

$$x_{t+2} = \begin{bmatrix} 10 \\ 9 \\ 8 \\ 7 \end{bmatrix}$$

what is the value of the asset, S_t, and what is its (time $t+2$) forward price?

(d) compute the forward state prices, q_k and re-compute the forward price of the asset, $F_{t,t+2}$.

5. Forward and futures prices

   (a) Given the following data (notation in PS ch6, section 6.4):

$$\mu_x = 4.605, \sigma_x = 0.15$$

$$\mu_\phi = -0.02, \sigma_\phi = 0.2$$

$$\rho(x, \phi) = -0.7$$

$$\sigma_b = 0.08, \rho(x, b) = 0.6$$

   compute the futures price and the forward price of the asset

   (b) (notation in PS ch6, section 6.5) Assume that the futures price of a zero-coupon bond is $H_{t,t+T} = 0.95$ and the forward price of the zero-coupon bond is $F_{t,t+T} = 0.955$. What is the futures rate, $h_{t,t+T}$? What is the forward rate, $f_{t,t+T}$?

Exercises: Hand in Dates

1. October 14, 10 a.m.
2. October 28, 10 a.m.
3. November 18, 10 a.m.
4. December 2, 10 a.m.
5. December 16, 10 a.m.

Each exercise counts for 4% of the course grade.

Exercises can be downloaded from rstapleton.com