BMAN 70381: Foundations of Finance Theory: Exercises

September 2013

1. Mean-variance portfolio analysis:

   Assume the following data for stocks 1 and 2:

   (a) Expected cash flow: $E(x_1) = 10$, Standard deviation: $\sigma_1 = 1.7$, Stock value: $S_1 = 6.4$, Number of shares: $n_1 = 300$.

   (b) Expected cash flow: $E(x_2) = 2.5$, Standard deviation: $\sigma_2 = 0.2$, Stock value: $S_2 = 2.0$, Number of shares: $n_2 = 150$. Correlation between the cash flows: $\rho_{1,2} = 0.4$.

   Assume that the risk-free rate of interest is $r_f = 0.05$.

   Assume a mean-variance utility function. Also, assume the following for investors 1 and 2: wealth, $w_1 = 12$, $w_2 = 9$. Risk parameter, $\lambda_1 = 0.3$, $\lambda_2 = 0.5$.

   (a) State the first order conditions for a maximum of the utility of investor 1.

   (b) Compute the optimal stock portfolio for investor 1 and the amount of borrowing/lending.

   (c) Use the 'Separation Theorem' to compute the stock demands and the borrowing/lending for investor 2.

   Note, given a matrix:

$$A = \begin{pmatrix} a, b \\ c, d \end{pmatrix}$$

   the inverse matrix is

$$A^{-1} = \begin{pmatrix} \frac{d}{ad-bc}, \frac{b}{bc-ad} \\ \frac{c}{bc-ad}, \frac{a}{ad-bc} \end{pmatrix}$$

2. The Black model

   Assume you wish to value a 1-year call option on a stock given:

   Stock price \$93

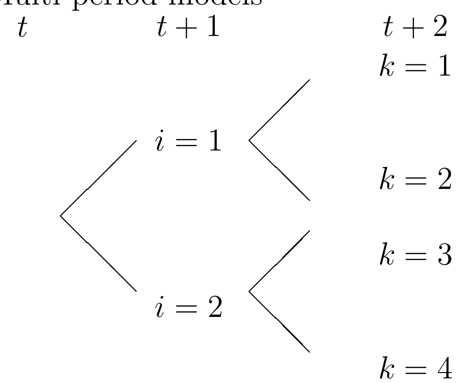
   Strike price \$109

   volatility 20%

   Risk free rate (continuously compounded) 6%

   What is the forward value of the option assuming a) no dividends on the stock? and b) a dividend yield of 2%? What is the spot value of the option in each case?

3. Multi-period models



Let the spot state prices, $q_i^* = q_i B_{t,t+1}$ and $q_k^* = q_k B_{t,t+2}$

$$q_i^* = \begin{bmatrix} q_{i=1}^* \\ q_{i=2}^* \end{bmatrix} = \begin{bmatrix} 0.43 \\ 0.5 \end{bmatrix}$$

$$q_k^* = \begin{bmatrix} q_{k=1}^* \\ q_{k=2}^* \\ q_{k=3}^* \\ q_{k=4}^* \end{bmatrix} = \begin{bmatrix} 0.16 \\ 0.18 \\ 0.18 \\ 0.18 \end{bmatrix}$$

If the payoffs on an asset at $t+1$ and $t+2$ are given by the vectors:

$$x_{t+1} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

$$x_{t+2} = \begin{bmatrix} 11 \\ 8 \\ 7 \\ 6 \end{bmatrix}$$

(a) Value the asset, S_t, using the Time-State Preference approach

(b) Value the asset, S_t, using the Rational Expectations approach (show all steps in the calculation)

(c) Explain why the two values are equal

4. Forward and futures prices

   (a) Given the following data (notation in PS ch6, section 6.4):

$$\mu_x = 4.6, \sigma_x = 0.15$$

$$\mu_\phi = -0.045, \sigma_\phi = 0.25$$

$$\rho(x, \phi) = -0.5$$

$$\sigma_b = 0.08, \rho(x, b) = 0.4$$

   compute the futures price and the forward price of the asset

   (b) (notation in PS ch6, section 6.5) Assume that the futures price of a zero-coupon bond is $H_{t,t+T} = 0.939$ and the forward price of the zero-coupon bond is $F_{t,t+T} = 0.946$. What is the futures rate, $h_{t,t+T}$? What is the forward rate, $f_{t,t+T}$?

Exercises: Hand in Dates

Hand in to the PGT office

1. October 9, 10 a.m.
2. November 6, 10 a.m.
3. November 20, 10 a.m.
4. December 4, 10 a.m.

Each exercise counts for 2.5% of the course grade.

Exercises can be downloaded from rstapleton.com